Variation of Entropy through a Shock Wave

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ALTHOUGH the velocity, pressure, temperature, and density vary monotonically through a shock wave, the entropy increases only at first, achieves a maximum within the shock structure, and then decreases to some final value. This nonmonotonic behavior of the entropy has been shown to be related to the heat conduction within the shock wave,[1] but this observation does not provide insight to its physical consequences, i.e., how does the heat conduction affect the shock structure such that the entropy decreases toward the rear of the wave. The purpose of this note is to present some results that connect the gasdynamics of the nonmonotonic entropy behavior to the heat conduction within the shock structure.

The entropy may be expressed as a function of the density and temperature as follows:

$$S - S_1 = C_v \ln\frac{T/\rho^{\gamma-1}}{T_1/\rho_1{}^{\gamma-1}} \tag{1}$$

where the subscripts refer to reference conditions. All symbols used in this analysis are standard notation. Using Eq. (1) and the continuity equation, $(d/dx)(\rho uA) = 0$, the density may be eliminated from Eq. (1) to give

$$\left\{\frac{dS/dx}{C_v} - (\gamma - 1)\frac{dA/dx}{A}\right\} = (\gamma - 1)\frac{du/dx}{u} + \frac{dT/dx}{T} \tag{2}$$

It is both advantageous and not unduly restrictive to assume that the total temperature of the flow is invariant. With this assumption, the energy equation may be written

$$C_p(dT/dx) + (udu/dx) = 0 \tag{3}$$

Equation (3) may be solved for dT/dx, and the result substituted into Eq. (2) to give

$$\left\{\frac{dS/dx}{C_v} - (\gamma - 1)\frac{dA/dx}{A}\right\} = (\gamma - 1)[1 - M^2]\frac{du/dx}{u} \tag{4}$$

where the relations $a^2 = \gamma RT$ and $R = C_p - C_v$ have been used.

This equation shows that during the course of any constant total temperature flow process in which the Mach number varies from some supersonic to some subsonic value (or the converse) the quantity within the braces must change sign. The equation therefore establishes the nonmonotonic entropy behavior where $(dA/dx) = 0$. It also shows that the magnitude of the entropy gradient at some point in any flow process depends upon the departure of the streamtube area variation from the isentropic streamtube area variation. This is a fundamental point with regard to the nonmonotonic entropy behavior within the shock wave (and to the entropy behavior in general).

Inspection of the subsonic region of the shock structure in light of the foregoing discussion shows why the entropy variation must necessarily be negative if nonzero. The streamtube area is invariant (for the case of a normal shock) while the streamtube area for constant entropy flow should increase. The constant area constraint causes a departure from isentropic conditions, and results in the entropy decrease toward the rear of the shock.

The roll of heat conduction in the nonmonotonic entropy behavior is determined as follows. The energy equation may be written

$$\rho u\frac{dE}{dx} + p\frac{du}{dx} - \frac{4}{3}\mu\left(\frac{du}{dx}\right)^2 - \frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0 \tag{5}$$

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This equation may be used in place of Eq. (3) to give the following expression:

$$\rho uT\left\{\frac{dS/dx}{C_v} - (\gamma - 1)\frac{dA/dx}{A}\right\} = \frac{1}{C_v}\left[\frac{4}{3}\mu\left(\frac{du}{dx}\right)^2 + \frac{d}{dx}\left(k\frac{dT}{dx}\right)\right] \tag{6}$$

It has been shown[1] that, at the front and rear points of the shock structure, the right-hand side of Eq. (6) will have the same sign as $(d/dx)[k(dT/dx)]$. Consequently, toward the rear of the shock structure, the quantity in the braces on the left will be negative for $k > 0$ and, hence, the entropy will be decreasing. In addition, Eq. (6) shows that for $k > 0$ and $dS/dx = 0$, $dA/dx > 0$. This means that the heat conduction causes a constant entropy streamtube to increase its area. It thus appears that the nonmonotonic entropy variation is due to a coupling of the heat conduction within the shock structure with the one-dimensional constraint on the compression process.

It should be noted that a rather complete treatment of the shock wave nonmonotonic entropy behavior exists in the literature.[2]

References

[1] Serrin, J. and Whang, Y. C., "On the entropy change through a shock layer," J. Aeronaut. Sci. **28,** 990 (1961).

[2] Morduchow, M. and Libby, P. A., "On the distribution of entropy within the structure of a normal shock wave," PIBAL Rept. 759 (August 1961).

Ionization of Trace Species in Slender Cone Boundary Layers

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Nomenclature

C^m = mass fraction of element m
C_m = mass fraction of species m
f = nondimensional stream function
F, G = Eq. (13)
I = Eq. (5)
k = forward ionization rate constant
K = Eq. (8)
l = $\rho\mu/\rho_w\mu_w$
M = molecular weight of mixture
r = radius of cone surface
Sc = Schmidt number
T = temperature
u, v = tangential, normal velocities in boundary layer
w = chemical mass generation rate
x, y = tangential, normal coordinate in boundary layer
Z = Eq. (13)
ϵ = Eq. (13)
η = $u_e r(2\xi)^{-1/2}\int_0^y \rho\, dy$
μ = viscosity
ρ = density
ξ = $\int_0^y \rho_w\mu_w u_e r^2 dx$

Subscripts

e = at edge of boundary layer
eq = equilibrium
max = maximum, or at maximum temperature
w = wall

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